Factorial function

The notation $n!$ means $1.2.3.\cdots.n$. Thus $5! = 1.2.3.4.5 = 120$.

Pascal's Triangle

(Left-justified)

n \ r	0	1	2	3	4	5	6	7
1	1	1						
2	1	2	1					
3	1	3	3	1				
4	1	4	6	4	1			
5	1	5	10	10	5	1		
6	1	6	15	20	15	6	1	
7	1	7	21	35	35	21	7	1

Observe

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

... ...

$(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

Pascal's triangle is also used in Permutations and Combinations.

Q. How many combinations are there by selecting two letters from five letters without order. With letters a,b,c,d,e we have the combinations

ab, ac, ad, ae, bc, bd, be, cd, ce, de 10 combinations - ab and ba are considered the same, etc.

In general the number of combinations made from n letters taken r at a time, without order is denoted by ${}_nC_r$ (or ${}^nC_r$) and is defined as ${}^nC_r = \frac{n!}{r!(n-r)!}$ Thus for the Q above, $n = 5$ and $r = 2$ so that the number of combinations is ${}^5C_2 = \frac{5!}{2!3!} = \frac{5.4}{1.2} = 10$.

Observe that by looking at Pascal's triangle in row marked n=5 and column marked r=2 to see that ${}^5C_2 = 10$.

For Permutations, where order does matter, the number of Permutations is ${}^nP_r = \frac{n!}{(n-r)!}$. For the example ${}^5P_2 = \frac{5!}{3!} = 20$. These permutations of two out of five letters, where order **does** matter, is

ab, ba, ac, ca, ad, da, ae, ea, bc, cb, bd, db, be, eb, cd, dc, ce, ec, de, ed.